

MATH1520AB 2021-22 Quiz 3 (week 5) Solution

Full marks: 10 marks

Time allowed: 15 minutes

1. Find the derivative of x^3 by using the definition of derivative (i.e. using first principle). (Hint: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ for any $a, b \in \mathbb{R}$.)

Answer. $(x^3)' = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 = 3x^2.$

2. Evaluate the derivative of the following functions with respect to x . (You can use all the results in lecture notes and do not have to find the derivatives using first principle.)

(a) $y = 3x^5 + 1$

(b) $y = \sqrt{x^2 + 1}$

(c) $y = e^x + 2^x + \log_3(x)$

Answer.

(a) $\frac{dy}{dx} = 3(5)x^4 + 0 = 15x^4.$ (Power rule + constant rule)

(b) $\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)^{-1/2} \frac{d(x^2 + 1)}{dx} = \frac{1}{2} \frac{2x}{\sqrt{x^2 + 1}} = \frac{x}{\sqrt{x^2 + 1}}.$ (Chain rule for the first equality)

(c) $\frac{dy}{dx} = \frac{d(e^x + e^{\ln 2 x} + \frac{\ln x}{\ln 3})}{dx} = e^x + \ln 2 e^{\ln 2 x} + \frac{1/x}{\ln 3} = e^x + \ln 2 2^x + \frac{1}{x \ln 3}.$ (Or you can use the formula in notes directly)